

# FUNDING SOCIAL SECURITY: THE TRANSITION IN A LIFE-CYCLE GROWTH MODEL

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## INTRODUCTION

Interest in policy proposals to gradually convert Social Security from pay-as-you-go (paygo) to full funding has been recently renewed. Under paygo, taxes on current workers directly finance benefits to current retirees without any accumulation of a fund. Under full funding, each generation of workers accumulates its own fund that it draws on during its own retirement. Full funding can be accomplished in two ways. Social Security can be privatized so that each individual accumulates a personal defined-contribution fund. Or Social Security can accumulate a single large fund (managed by private firms under contract with the Social Security Administration) that generates investment income as the primary source of financing defined benefits. The analysis in this paper applies to the transition from paygo to full funding whether accomplished through many individual funds or a single collective fund.<sup>1</sup>

Too often the discussion of funding Social Security simply compares steady states: an economy with paygo Social Security and an economy with funded Social Security. The question is posed: Would it be better for a person to be born into an economy with paygo Social Security or an economy with funded Social Security? The literature on life-cycle growth models [Seidman, 1986; Auerbach and Kotlikoff, 1987] suggests that, for a wide range of empirically plausible parameter values, a person born into a funded Social Security economy would enjoy higher lifetime utility than a person born into a paygo Social Security economy. Why? Because in these models the steady state of the funded Social Security economy would have a higher capital stock, produce higher output, and generate a higher individual lifetime consumption path. Or another question is posed: In which economy would a worker receive a higher rate of return on her Social Security "saving" (her contribution in the funded system or her payroll tax in the paygo system)? Once again, if the comparison is between steady state economies, the answer is likely to be that the return would be higher in the funded Social Secu-

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rity economy. Why? Because the return in the funded Social Security economy equals the marginal product of capital, whereas the return in the paygo economy equals the growth rate of the economy (the sum of labor force growth and the rate of technological progress) [Seidman, 1983; 1999], and for a wide variety of empirically plausible parameter values, the marginal product of capital exceeds the growth rate of the economy.<sup>2</sup> So evidence suggests that transforming our paygo Social Security steady state into a funded Social Security steady state might be socially desirable if we could wave a wand and do so instantly and costlessly: it would raise the lifetime utility of the typical person, and achieve a higher rate of return for each worker on her Social Security saving.

But we cannot wave a wand. The United States, and many other countries, currently have a paygo Social Security economy. To achieve the funded Social Security economy's steady state, they must embark on a long and arduous transition path that involves the accumulation of capital stock required to raise future consumption and utility. During what may be a lengthy transition period, consumption will be lower than if otherwise would have been, almost certainly imposing a loss of rest-of-life utility on some age cohorts.

Most economists recognize the importance of the transition path [Aaron and Reischauer, 1998; Bosworth, 1996; Diamond, 1997; Gramlich, 1996]. But recognition is only a first step toward making a sensible policy decision. We must also assess the magnitude of the transitional losses, the allocation of these losses to different age cohorts who will live through the transition, and the quantitative consequence of varying the speed of conversion. Conversion may well be judged worthwhile if the transitional losses are small and brief, but not so if they are large and prolonged. Such a judgement requires quantitative analysis: how large are the transitional losses, and how are they allocated across different age cohorts? Moreover, how does varying the speed of conversion affect the magnitude of the losses?

This paper addresses these quantitative questions. This paper uses a life-cycle growth model, with plausible parameter values based on econometric studies, to investigate the impact of gradually converting the financing of Social Security from paygo to full funding. It differs from earlier studies, which include studies by Auerbach and Kotlikoff [1987], Kotlikoff [1996; 1998], Feldstein and Samwick [1997; 1998], and Kotlikoff, Smetters, and Walliser [1998], by focusing on the sensitivity of the pattern of cohort losses and gains to the speed of phasing out paygo Social Security and to plausible changes in key parameter values.

Feldstein and Samwick [1997] use a partial equilibrium framework to simulate the transition path to Social Security with full funding. They address the concern about the burden borne by the transition generation because they must continue to pay for existing retirees while also saving for their own retirement. According to their quantitative analysis, the burden will be small, not large. Moreover, they find that the loss to middle-aged and older workers is less than the gain to their children, so that virtually all nuclear families are net gainers.<sup>3</sup> In two papers, Kotlikoff [1996; 1998] simulates the transition path to Social Security with full funding using a version of the life-cycle growth model described in Auerbach and Kotlikoff [1987]. Unlike Feldstein and Samwick, Kotlikoff uses a general equilibrium framework: wages and

interest rates are affected by the capital accumulation that results from funding. He finds that the maximum loss in rest-of-life utility for any one age cohort is small.<sup>4</sup> Kotlikoff, Smetters, and Walliser [1998] simulate the transition path to Social Security with funding using an enhanced version of the Auerbach/ Kotlikoff life-cycle growth model that includes heterogeneous earning classes within each age cohort. They also find that the largest loss is small.<sup>5</sup>

We use a general equilibrium life-cycle growth model to investigate several issues not studied by Kotlikoff and his colleagues. First, we examine perhaps the central policy issue: the speed of phasing out paygo. Does the speed significantly affect the pattern of cohort gains and losses? We find that phase-outs shorter than 45 years impose losses on particular age cohorts so large that they would be most likely judged politically unfeasible. We therefore focus our attention on four phase-out options: 45, 60, 75, and 90 years. How much do these four phase-out options differ? Must a phase-out take more than three generations to generate an acceptable pattern of cohort gains and losses? Should the beginning of the phase-out be delayed several years after the conversion is announced? Kotlikoff [1996, 10] says that one of the “three key decisions” in funding Social Security is “how fast to phase-out benefits.” However, he presents results only for a two-generation (55-year) phase-out (with a delay of 10 years, and a duration of 45 years) and an abrupt cold-turkey termination. A central focus of our study is a comparison of four alternative phase-out speeds that policy makers are likely to consider: 45, 60, 75, and 90 years.

Second, we examine the sensitivity of the pattern of cohort gains and losses during the phase-out to six key parameters. The first two parameters characterize the lifetime utility function of the individual: the intertemporal elasticity of substitution<sup>6</sup> and the rate of pure time preference. The intertemporal elasticity of substitution indicates how much the individual will shift consumption between ages of life in response to changes in the interest rate. The rate of pure time preference indicates how much the individual favors consumption at younger ages over consumption at older ages (for a given interest rate). The third parameter indicates the length of retirement, and the fourth parameter, the growth rate of the labor force. The last two parameters characterize the production process: the capital elasticity in the Cobb-Douglas production function and the growth rate of technological progress.

It is crucial to emphasize one feature of this paper. In our simulations, there are no intergenerational side transfers: age cohorts that lose must bear the loss without compensation, just as age cohorts that gain enjoy the full gain. We do not investigate the question of Pareto optimality: Would it be possible for the gainers to compensate the losers so that everyone is better off? Although this question has been addressed by others—for example, Auerbach and Kotlikoff [1987] consider hypothetical lump-sum redistributions among different age cohorts—in practice such compensation will not take place, and our paper tries to impress this point by focusing on the magnitude of the gains or losses to each age cohort generated by a particular conversion speed. As has been pointed out [Cordes and Weisbrod, 1985], converting a program gradually rather than abruptly is really a form of implicit compensation; moreover, making the conversion gradual in itself generates incentives for participants that may differ significantly from the incentives under an abrupt conversion. Our model assumes

that each economic agent has perfect foresight of the transition path under gradual conversion, so that our model simulations fully capture the response to the incentives generated by such gradual conversion. It should also be noted that, like Kotlikoff [1996; 1998], we focus on a gradual phase-out that begins with a delay (postponement). Our base case of a 90-year phase-out begins with a delay of 15 years, and then has a duration of 75 years; we compare results from this phase-out to a 90-year phase-out with no delay. The pros and cons of including a delay in a gradual phase-out are discussed by Feldstein [1976], Zodrow [1985], and Cordes and Weisbrod [1985].

It is important to acknowledge that we do not attempt to address the difference between accumulating capital through a single collective fund (the Social Security trust fund) rather than through individual private accounts. Rather, we emphasize the similarity: In both cases, funds are invested in real capital instead of being paid out immediately to retirees; hence, in both cases, the rate of return equals the marginal product of capital. But there may be important differences. A person may save more if she controls her own saving in her own individual account or she may save more if she has the opportunity to spread risk by pooling her savings with many others' through a single large collective fund. A person may work more with the motive of contributing to her own individual account, or she may work more if she has the opportunity to contribute to a large fund that spreads risk. In order to focus on capital accumulation, we simplify by suppressing any labor supply effects. Our model, therefore, does not attempt to examine these interesting questions.

In the next section we compare the paygo and funded steady states in our life-cycle growth model. We then analyze the response of the economy to a gradual conversion of financing from paygo to funding. All mathematical derivations and formulas are given in the Appendix.

## THE STEADY STATE WITH PAYGO

This section describes the steady state of the economy with paygo Social Security financed by a payroll (wage) tax with rate  $t_p$  (the subscript "p" indicates paygo). Government spending consists of Social Security benefits, and is financed by a wage (payroll) tax.<sup>7</sup> With paygo, annual benefits equal payroll taxes so there is no saving by Social Security, and national saving equals aggregate individual saving.

In the model the number of new entrants grows at rate  $n$ , and workers, retirees, and population each grow at rate  $n$ . Let  $\omega$  be the worker/retiree ratio.<sup>8</sup> Labor-augmenting technology progresses at rate  $g$ , so effective labor per worker grows at rate  $g$ , and each person's wage  $w$  grows at rate  $g$  over the work-life. We assume that each worker of any age  $t$  receives the same wage  $w$  and pays the same tax  $t_p w$  in year  $v$ , and that under paygo Social Security each retiree of any age  $t$  receives the same benefit  $B_p$  in year  $v$ .<sup>9</sup>

Let  $B_p/w$  be "the replacement rate," the ratio of the benefit per retiree to the wage per worker in year  $v$ . Since paygo Social Security has an annually balanced budget (revenue from workers equals benefits to retirees), in any year  $v$ ,  $B_p/w = t_p \omega$ . For example, if  $t_p = 15$  percent and  $\omega = 3$ , then  $B_p/w = 45$  percent. Just as the wage grows at rate  $g$  over a person's work-life, so the benefit grows at rate  $g$  over a person's retirement.

Total effective labor  $E$ , capital  $K$ , and output  $Y$  grow at the discrete period rate  $n + g + ng$ ; capital per effective labor ( $k \equiv K/E$ ) and output per effective labor ( $y \equiv Y/E$ ) are constant. Depreciation is assumed to be zero so that aggregate individual saving  $S$  equals the increase in the capital stock. Production is Cobb-Douglas with the capital exponent  $\alpha$ , and each factor is paid its marginal product: the interest rate  $r$  equals the marginal product of capital, and the wage per effective labor  $w_e$  equals the marginal product of effective labor; in the steady state,  $r$  and  $w_e$  are constant (the wage per worker  $w$  grows at the rate of labor-augmenting technical progress  $g$ ). Since the marginal product of capital decreases when  $k$  increases, the greater is  $k$ , the smaller is  $r$ .

Each person chooses a consumption path from the age of entry to the end of life to maximize an isoelastic utility function ( $\sigma$  is the intertemporal elasticity of substitution and  $\rho$  is the discount rate) subject to the lifetime budget constraint. Aggregating over all age cohorts in year  $v$  yields a ratio of aggregate consumption to aggregate labor income. Using the marginal productivity assumption, we are able to derive a formula which yields the steady-state interest rate  $r$  of the economy. From  $r$ , we get capital per effective labor  $k$  and the saving rate  $s$ . Each person works from 1 to  $R$  and retires from  $R + 1$  to  $J$ . For example, if a person enters the economy after age 20 and retires after age 65 through age 80, then  $R = 45$  and  $J = 60$ . Given the parameter values  $n = 0$  percent,  $g = 2$  percent,  $\alpha = 0.3$ ,  $\sigma = 0.5$ ,  $\rho = 1.5$  percent,  $R = 45$ ,  $J = 60$ , and  $t_p = 15$  percent, we find from the formula derived in the Appendix that  $r = 8.1$  percent; then  $k = 6.5$  and  $s = 7.4$  percent. We will use this set of parameter values for our base case.<sup>10</sup>

It can be shown [Seidman, 1983; 1999] that, under paygo Social Security, a worker's rate of return on Social Security "saving" (payroll tax) equals  $(n + g + ng)$ ; with our base parameter values of  $n = 0$  percent and  $g = 2$  percent, the rate of return is 2 percent.<sup>11</sup>

## THE STEADY STATE WITH FUNDING

With funding, each person has two wealth accounts, a private account  $A$  and a Social Security account  $F$ , which each yield the same interest rate  $r$  equal to the marginal product of capital (mpk). Thus, funded Social Security yields a return equal to  $r = \text{mpk}$ . (As noted above, paygo Social Security yields a return equal to  $n + g + ng$ .) We assume each person treats the two accounts,  $A$  and  $F$ , as perfect substitutes [Kotlikoff, 1996; 1998]. Just as deposits into, and withdrawals from, the  $A$  account do not appear in a person's lifetime budget constraint equation, so deposits into and withdrawals from the  $F$  account do not appear. The person chooses a lifetime consumption path that implies a path for wealth  $H$ . The allocation of wealth  $H$  between accounts  $F$  and  $A$  depends on the payroll tax rate  $t_f$  for the Social Security account  $F$  (the subscript " $f$ " indicates funding). A higher  $t_f$  causes more wealth to be allocated to  $F$  and therefore less to  $A$ .

In the model where each person maximizes lifetime utility in a perfect capital market (with unlimited borrowing), the steady state with funded Social Security is the same as the steady state with no Social Security. It follows that the steady-state  $r$  with funding is obtained by setting  $t_p = 0$  percent and that  $t_f$  has no effect on the

steady-state equilibrium. For example, with the above parameter values, Social Security with funding yields  $r = 6.5$  percent; then  $k = 8.9$  and  $s = 9.3$  percent (compared with the paygo results  $r = 8.1$  percent,  $k = 6.5$  and  $s = 7.4$  percent).

With funded Social Security, a worker's rate of return on Social Security saving equals  $r$ , which equals the marginal product of capital; hence, with our base parameter values, the rate of return is 6.5 percent, much larger than the rate of return with paygo Social Security of 2 percent. More generally, as long as  $r$  exceeds  $n + g + ng$ , the return with funded Social Security exceeds the return with paygo.

Table 1 uses base parameter values to compare the experience of an individual over a lifetime in the two steady states: paygo and funded. For each steady state, the table presents the values of five variables at selected ages: wage income ( $W$ ), total (wage plus capital) income ( $Y$ ), consumption ( $C$ ), saving ( $S$ ), and wealth ( $H = A + F$ ). For paygo, a sixth variable is presented: net income ( $Y_n$ ). For ages 21 through 65,  $Y_n$  equals  $Y$  minus the paygo wage tax; for ages 66 through 80,  $Y_n$  equals  $Y$  plus the paygo benefit. We note several points. First, from the  $W$  columns, at each age the funded wage is greater than the paygo wage because the funded steady state has a higher capital per effective labor  $k$ , and therefore a higher marginal product of labor, than the paygo steady state (along both paths, the wage grows at the rate of labor-augmenting technical progress,  $g = 2$  percent; at any age, the funded wage is greater). Second, from the  $C$  columns, the individual maximizes lifetime utility by choosing a consumption path that grows at a constant rate from ages 21 through 80.<sup>12</sup> Thus, with funded Social Security, the individual begins (age 21) with a higher consumption (0.88 vs 0.64), but consumption then grows more slowly over the life cycle. Third, through higher saving during working years, the individual accumulates more wealth in the funded steady state; at age 66 when retirement begins and wealth is at its peak, wealth is 62 percent more (27.99 vs 17.23) in the funded steady state.

While the choice of the replacement rate with funding does not affect the steady state of this model (with every agent maximizing lifetime utility in a perfect capital market), policy makers and the public are likely to regard the replacement rate as important in the actual economy. We therefore assume that policy makers maintain the paygo replacement rate. For example, with the above parameter values including  $t_p = 15$  percent, the required tax rate with funding  $t_f$  is only 3.6 percent.<sup>13</sup> By comparison, using a partial equilibrium model, Feldstein and Samwick [1998] estimate that with a 5.4 percent real return, a  $t_f$  of 3.3 percent is required to replace a paygo rate of 12.4 percent. Thus, with funding, the same replacement rate is achieved by a much lower payroll tax rate.<sup>14</sup>

Suppose a person could live his entire life in either the paygo steady state or the funded steady state. In which steady state would the person enjoy higher lifetime utility, and how much higher would it be? Let  $C_f$  be the person's consumption path in the funded steady state,  $C_t$  be the consumption path in the paygo steady state, and  $\delta$  be the percentage increase in the paygo path  $C_t$  that would achieve the same utility as the person achieves with funded path  $C_f$ . If  $\delta$  is positive, the person would have a higher utility in the funded steady state. For our base parameter values,  $\delta$  equals 19.2 percent, so that the person's lifetime utility would be 19.2 percent higher in the funded steady state.



**TABLE 1**  
**Individual Life Time Paths in the Paygo and Funded Steady States**

Age	Paygo Steady State						Funded Steady State				
	<i>W</i>	<i>Y</i>	<i>Y<sub>n</sub></i>	<i>C</i>	<i>S</i>	<i>H</i>	<i>W</i>	<i>Y</i>	<i>C</i>	<i>S</i>	<i>H</i>
21	0.93	0.93	0.79	0.64	0.15	0.00	1.02	1.02	0.88	0.14	0.00
22	0.95	0.96	0.82	0.66	0.15	0.15	1.04	1.05	0.90	0.15	0.14
30	1.11	1.24	1.07	0.85	0.22	1.60	1.22	1.33	1.09	0.24	1.65
40	1.35	1.70	1.49	1.17	0.33	4.26	1.49	1.80	1.38	0.42	4.77
50	1.65	2.30	2.06	1.59	0.46	8.11	1.81	2.47	1.75	0.72	10.14
60	2.01	3.09	2.79	2.18	0.61	13.39	2.21	3.47	2.23	1.24	19.41
65	2.22	3.55	3.22	2.55	0.67	16.56	2.44	4.15	2.51	1.64	26.36
66	0.00	1.39	2.41	2.63	-0.22	17.23	0.00	1.81	2.57	-0.76	27.99
70	0.00	1.27	2.38	2.98	-0.61	15.81	0.00	1.57	2.83	-1.26	24.26
80	0.00	0.20	1.55	4.08	-2.53	2.53	0.00	0.22	3.60	-3.38	3.38

Base Parameter Values:  $n = 0$  percent,  $g = 2$  percent,  $\alpha = 0.3$ ,  $\sigma = 0.5$ ,  $\rho = 1.5$  percent, Last Working Age = 65 ( $R=45$ ), Last Retirement Age = 80 ( $J = 60$ ). Paygo Steady State ( $t_p = 15$  percent,  $B/w = 45$  percent):  $r = 8.1$  percent,  $s = 7.4$  percent. Funded Steady State:  $r = 6.5$  percent,  $s = 9.3$  percent,  $t_f = 3.6$  percent,  $k_f/k = 38$  percent.

## SIMULATION OF A PHASED CONVERSION FROM PAYGO TO FUNDING

The fundamental question is whether it is possible to convert from paygo to funding without imposing substantial losses on the current working population. It is often noted that current workers would have to continue to support current retirees while also saving for their own retirement, and concern is expressed that these workers might therefore suffer large losses. It is imperative to examine the size and pattern of losses during the transition.

We analyze the response of the economy to a gradual conversion of financing from paygo to funding.<sup>15</sup> In each year  $v$ , a person of age  $t$  chooses a rest-of-life consumption path to maximize an isoelastic rest-of-life utility function ( $\sigma$  is the intertemporal elasticity of substitution and  $\rho$  is the discount rate) subject to a rest-of-life budget constraint, thereby choosing  $C_t(v)$  and  $S_t(v)$ , the saving that will be channeled into the individual's two accounts: a Social Security account  $F$  and a private account  $A$ . Although funding is phased in as paygo is phased out, the  $t_f(v)$  schedule has no effect on the path of the economy, just as  $t_f$  has no effect on the steady state of the economy.<sup>16</sup>

We assume that each person has perfect foresight of the path of the economy, and hence, perfect foresight of his rest-of-life budget constraint. In particular, we assume that in each year each individual accurately projects the future path of his own wage and the interest rate, and accurately incorporates the scheduled phase-down of paygo Social Security.<sup>17</sup> We are able to derive an equation that yields the consumption that

a person age  $t$  will choose. The saving of a person age  $t$  in year  $v$ ,  $S_t(v)$ , equals the person's income minus consumption. Summing over all persons yields aggregate saving  $S(v)$ ; the capital stock in year  $v + 1$ , therefore, is  $K(v + 1) = K(v) + S(v)$ , and capital per effective labor in year  $v + 1$  is  $k(v + 1) \equiv K(v + 1)/E(v + 1)$ . Because each factor is paid its marginal product, from  $k(v + 1)$  we obtain  $r(v + 1)$  and  $w_e(v + 1)$ , where  $w_e$  is the wage per effective labor. Each person's saving,  $S_t(v)$ , is divided between two accounts: a Social Security account  $F$  and a private account  $A$ ; saving in each account earns a rate of return  $r$  (equal to the marginal product of capital). The amount of saving channeled into account  $F$  is determined by  $t_p$ , the payroll tax rate for funded Social Security; the residual saving is channeled into account  $A$ . Withdrawals from Social Security account  $F$  in retirement are called Social Security "benefits"  $B$ , and determine the replacement rate  $B/w$  with funded Social Security. In the simulations, each person's  $F$  account and  $A$  account are exactly exhausted at the end of retirement.

Let  $\delta$  be the percentage rest-of-life gain of a person age  $t$  in year 0. We compute  $\delta$  as follows. Let  $C_i$  be the actual consumption path over a person's rest-of-life and  $\bar{C}_i$  be the rest-of-life path the person would have followed had the economy remained in the paygo steady state.  $\delta$  is the percentage increase in the steady-state path  $\bar{C}_i$  that would achieve the same utility as the person actually achieves with path  $C_i$ . If  $\delta$  is negative, a person loses from the phased conversion of paygo to funded Social Security; if  $\delta$  is positive, a person gains. We compute  $\delta$  with an iterative procedure. For our base parameter values, we find that as the transition evolves, and we compute  $\delta$  for cohorts born further into the future,  $\delta$  approaches 19.2 percent, the value we reported above when we compared lifetime utility in the funded steady state and the paygo steady state. Hence, each person born in the distant future will be 19.2 percent better off if the economy has funded Social Security than if the economy has paygo Social Security. Below we provide information on how long it takes for  $\delta$  to approach its steady-state value of 19.2 percent.

Before presenting our results in detail in the tables below, it is useful to highlight one point: Like Feldstein and Samwick [1997], Kotlikoff [1996;1998], and Kotlikoff, Smetters, and Walliser [1998], we find that the maximum loss to any age cohort under a *gradual* conversion is small. However, in our model, it is necessary to convert more slowly than Kotlikoff (90 years instead of his 55 years) in order to keep the maximum loss the same as he reports—roughly 2 percent. With empirically plausible parameter values, we find that for a gradual three-generation (90 year) conversion, the maximum loss in rest-of-life utility for any age cohort is only 1.6 percent. However, we also find that faster conversions raise the maximum loss in rest-of-life utility. For example, for a one-and-a-half generation (45 year) conversion, the maximum loss is 4.2 percent. Still more rapid conversions would raise the maximum loss to levels that would likely be judged politically unfeasible.

Table 2 shows the consequences of a three-generation (90-year) phase-out of paygo for our base parameter values:  $n = 0$  percent,  $g = 2$  percent,  $\alpha = 0.3$ ,  $\sigma = 0.5$ ,  $\rho = 1.5$  percent,  $R = 45$ ,  $J = 60$ , and  $t_p = 15$  percent.<sup>18</sup> With a three-generation phase-down,  $t_p$  begins at 15 percent and is linearly reduced to 0 percent in year 90. Consider the right column of Table 2 where the delay is 15 years, and the  $t_p$  phase-down has a duration of 75 years so  $t_p$ , beginning at 15.0 percent, is reduced 0.2 percent per year during the



**TABLE 2**  
**Sensitivity to Alternative  $t_p$  Delay**

$t_p$ Delay $t_p$ Duration	90-Year (Three-Generation) Paygo Phase Out with $t_f$ Duration = 45		
	0 90	10 80	15 75
Max Loss (Age)	3.3% (61)	2.0% (49)	1.6% (43)
Loss Age Range	80 to 30	69 to 22	65 to 17
Gain ( $\delta$ ) at Age 60	-3.3%	-1.1%	-0.4%
Gain ( $\delta$ ) at Age 30	-0.1%	-0.9%	-1.1%
Gain ( $\delta$ ) at Age 0	5.4%	3.8%	2.9%
Max $B/w$ (Age)	51% (-7)	53% (-5)	54% (-4)
Min $B/w$ (Age)	33% (38)	36% (37)	38% (36)
Max Tax	14.8%	15.8%	16.2%

Base Parameter Values:  $n = 0$  percent,  $g = 2$  percent,  $\alpha = 0.3$ ,  $\sigma = 0.5$ ,  $\rho = 1.5$  percent, Last Working Age = 65 ( $R = 45$ ), Last Retirement Age = 80 ( $J = 60$ ). Paygo Steady State ( $t_p = 15$  percent,  $B/w = 45$  percent):  $r = 8.1$  percent,  $s = 7.4$  percent. Funded Steady State:  $r = 6.5$  percent,  $s = 9.3$  percent,  $t_f = 3.6$  percent,  $k_f/k = 38$  percent.

phase-down. The maximum loss for any age cohort (1.6 percent) occurs for a person who is age 43 (in the year the phase-down is enacted). The 15-year delay in phasing down  $t_p$  keeps older persons (who are above age 65 in the year of enactment) from experiencing a loss. Losses then rise monotonically as age in year 0 falls until the maximum loss is reached; the losses then fall monotonically thereafter.<sup>19</sup> Everyone from age 65 through 17 (in year 0) experiences a small loss. Once the gains begin (under age 17), they rise monotonically as age in year 0 falls. Due to the 15-year delay, an older worker age 60 experiences only a very small loss ( $\delta = -0.4$  percent), a younger worker age 30 experiences a small loss ( $\delta = -1.1$  percent), and the younger worker's newborn child age 0 enjoys a rest-of-life gain ( $\delta = 2.9$  percent).<sup>20</sup> Although not reported in the table, the  $\delta$  for cohorts yet to be born continues to rise into the future until it approaches 19.2 percent.

Although the phase-in of funded Social Security with tax rate  $t_f$  is assumed (as explained earlier) to have no effect on the path of the economy,  $t_f$  does affect the replacement rate—the ratio of the Social Security benefit to the wage ( $B/w$ ). Throughout this paper, we assume that the  $t_f$  phase-in is half as long as the paygo phase-out, so in Table 2 where the paygo phase-out is 90 years, the  $t_f$  phase-in is 45 years:  $t_f$  begins at 0 percent, increases 0.08 percent per year, and becomes fixed permanently at 3.60 percent after 45 years. As a consequence of this 45-year  $t_f$  phase-in, in the right column of the table, the maximum replacement rate ( $B/w$ ) of any cohort is 54 percent (for cohort age-4), and the minimum replacement rate ( $B/w$ ) of any cohort is 38 percent (for cohort age 36); the paygo replacement rate with  $t_p = 15$  percent is 45 percent. The maximum combined tax rate ( $t_p + t_f$ ) along the transition path is 16.2 percent which occurs at the end of the 15 year delay (the year before the phase-out of  $t_p$  begins), when  $t_p$  is still 15 percent and  $t_f$  is 1.2 percent (one third of its permanent value of 3.6 percent).

**TABLE 3**  
**Sensitivity to Alternative Paygo Phasing Speeds**

	45-Year Paygo Phase Out	60-Year Paygo Phase Out	75-Year Paygo Phase Out	90-Year Paygo Phase Out
$t_p$ Delay	15	15	15	15
$t_p$ Duration	30	45	60	75
Max Loss (Age)	4.2% (43)	2.8% (43)	2.0% (43)	1.6% (43)
Loss Age Range	69 to 20	67 to 17	66 to 17	65 to 17
Gain ( $\delta$ ) at Age 60	-1.0%	-0.7%	-0.5%	-0.4%
Gain ( $\delta$ ) at Age 30	-2.8%	-1.9%	-1.4%	-1.1%
Gain ( $\delta$ ) at Age 0	9.2%	5.8%	3.7%	2.9%
Max $B/w$ (Age)	49% (1)	50% (-5)	50% (-12)	54% (-4)
Min $B/w$ (Age)	22% (32)	32% (32)	36% (35)	38% (36)
Max Tax	17.4%	16.8%	16.4%	16.2%

Base Parameter Values:  $n = 0$  percent,  $g = 2$  percent,  $\alpha = 0.3$ ,  $\sigma = 0.5$ ,  $\rho = 1.5$  percent, Last Working Age = 65 ( $R = 45$ ), Last Retirement Age = 80 ( $J = 60$ ). Paygo Steady State ( $t_p = 15$  percent,  $B/w = 45$  percent):  $r = 8.1$  percent,  $s = 7.4$  percent. Funded Steady State:  $r = 6.5$  percent,  $s = 9.3$  percent,  $t_f = 3.6$  percent,  $k_f/k = 38$  percent.

The three columns of Table 2 differ according to the delay before the phase-out of  $t_p$  begins. The left column reports the case of no delay; the middle column, a 10-year delay; and the right column, a 15-year delay. In all three cases,  $t_p$  reaches 0 percent in 90 years after the enactment of the planned phase-out. We assume that economic agents change behavior in year 0—the year of enactment—in anticipation of the scheduled phase-down that is delayed (for example, 10 or 15 years).

With a three-generation phase-out of paygo, how large is the maximum loss to any age? In the right column with a 15-year delay, the maximum loss is only 1.6 percent (at age 43), whereas in the left column with no delay, the maximum loss is 3.3 percent (at age 61). Thus, delaying the phase-down by 15 years cuts the maximum loss and reduces the age cohort that experiences that loss. However, this delay reduces the gain of the newborn (age 0) from 5.4 percent to 2.9 percent.

Table 3 shows the sensitivity to alternative speeds for phasing out paygo Social Security (with a 15-year delay) and phasing in Social Security with funding. The first column is a 45-year phase-out of  $t_p$ ; the second column, a 60-year; the third column, a 75-year; and the fourth column, a 90-year (three-generation) phase-out. In each case, the  $t_p$  delay is 15 years, and the  $t_f$  phase-in is half the  $t_p$  phase-out. Note that the right column in Table 3 is identical to the right column in Table 2. The advantage of the slower 90-year phase-out over the faster phase-outs (75, 60, and 45) is that the maximum loss for any age is smaller (1.6 percent vs 2.0 percent, 2.8 percent, 4.2 percent).<sup>21</sup> We also ran faster conversions (less than 45 years) not shown in the table. As expected, the maximum loss increases the faster the conversion. It seems likely that faster conversions, which would impose losses well above 4.2 percent on particular

**TABLE 4**  
**Sensitivity to Alternative Parameter Values, 90-Year (Three-Generation)**  
**Paygo Phase-Out:  $t_p$  Delay = 15,  $t_p$  Duration = 75,  $t_f$  Duration = 45**

	Base	$n = 1\%$	$g = 1\%$	$\alpha = 0.35$	$\sigma = 0.25$	$\rho = 3.0\%$	$J = 55$
Max Loss (Age)	1.6% (43)	1.8% (43)	1.6% (42)	1.3% (44)	0.7% (49)	1.3% (45)	1.1% (43)
Loss Age Range	65 to 17	65 to 14	65 to 15	65 to 20	66 to 21	65 to 22	59 to 20
Gain ( $\delta$ ) at Age 60	-0.4%	-0.4%	-0.4%	-0.3%	-0.3%	-0.4%	-0.0%
Gain ( $\delta$ ) at Age 30	-1.1%	-1.3%	-1.2%	-0.8%	-0.0%	-0.6%	-0.7%
Gain ( $\delta$ ) at Age 0	2.9%	2.5%	2.6%	3.3%	4.2%	3.6%	3.1%
Paygo $r$	8.1%	8.8%	6.6%	8.7%	13.8%	9.4%	9.3%
Funded $r$	6.5%	7.0%	5.2%	7.2%	10.6%	7.8%	7.6%

Base Parameter Values:  $n = 0$  percent,  $g = 2$  percent,  $\alpha = 0.3$ ,  $\sigma = 0.5$ ,  $\rho = 1.5$  percent, Last Working Age = 65 ( $R = 45$ ), Last Retirement Age = 80 ( $J = 60$ ).

age cohorts, would be judged politically unfeasible. We therefore confine our table to speeds of conversion ranging from 45 to 90 years.

The disadvantage of the slower 90-year phase-out is that the gains for a newborn emerge more slowly. For a newborn (age 0), the gain is 2.9 percent vs 3.7 percent, 5.8 percent, 9.2 percent (recall that in the distant future the gain of a newborn will be 19.2 percent). The effect of a slower phase-out is to smooth the gains and losses across cohorts. As we move from left to right, the losses for ages 60, 43 (maximum loss age), and 30 are smaller, and the gains for age 0 are also smaller.

With the 90-year phase-out, the range for the replacement rate  $B/w$  of different age cohorts is narrow and symmetrical around the paygo value of 45 percent (54 percent to 38 percent), whereas with the 45-year phase-out, the range is wide and asymmetrical (49 percent to 22 percent). With the 90-year phase-out, the maximum combined tax rate ( $t_p + t_f$ ) is 16.2 percent, whereas with the 45-year phase-out it is 17.4 percent. The choice of how many generations to include in the paygo phase-out, and whether to delay it, matters significantly: with the right choice, transitional losses can be kept low.

Table 4 shows the sensitivity of the pattern of cohort gains and losses to changes in parameter values. Beginning from the base parameter values (column 1), we vary the value of one parameter at a time in order to assess the sensitivity to that parameter. Although some empirically reasonable parameter changes cause a substantial change in the paygo steady-state interest rate  $r$  and the funded steady-state  $r$ , none of the changes have much effect on the pattern of cohort gains and losses. For example, with an empirically-plausible reduction<sup>22</sup> in the value of the intertemporal elasticity of substitution  $\sigma$  from 0.50 to 0.25, the maximum loss of any cohort changes from 1.6 percent to 0.7 percent, and the gain of the cohort with age 0 changes from 2.9 percent to 4.2 percent. Thus, our conclusions concerning the pattern of cohort gains and losses are not sensitive to empirically plausible changes in key parameter values.

Finally, it is useful to compare our results with Kotlikoff's [1998]. Before presenting the details, we summarize the main conclusions from the comparison. In our model,

**TABLE 5**  
**Model Comparisons: Percentage Change in Output and Interest Rate**  
**Relative to Paygo Steady State**  
**55-Year Paygo Phase Out:  $t_p$  Delay=10,  $t_p$  Duration=45**

Year	Kotlikoff [1998] Paygo $r = 9.1\%$	Kotlikoff Parameter Values <sup>a</sup> Paygo $r = 9.0\%$	Base Parameter Values <sup>b</sup> Paygo $r = 8.1\%$
<b>Output</b>			
5	0.5%	0.3%	0.3%
10	0.5%	0.8%	0.7%
25	4.8%	3.2%	2.6%
150	17.1%	13.1%	9.8%
<b>Interest Rate</b>			
5	-0.6%	-0.9%	-0.6%
10	-1.1%	-2.3%	-1.6%
25	-2.6%	-9.0%	-5.8%
150	-25.3%	-30.9%	-19.6%

a. Kotlikoff Parameter Values:  $n = 1$  percent,  $g = 0$  percent,  $\alpha = 0.25$ ,  $\sigma = 0.25$ ,  $\rho = 1.5$  percent, Last Working Age = 65 ( $R = 45$ ), Last Retirement Age = 75 ( $J=55$ ).

b. Base Parameter Values:  $n = 0$  percent,  $g = 2$  percent,  $\alpha = 0.3$ ,  $\sigma = 0.5$ ,  $\rho = 1.5$  percent, Last Working Age = 65 ( $R = 45$ ), Last Retirement Age = 80 ( $J = 60$ ).

it is necessary to convert more slowly than Kotlikoff, 90 years instead of his 55 years, in order to keep the maximum loss the same as he reports—roughly 2 percent. For our 90 year conversion (15-year delay and 75-year duration), the maximum loss is 1.6 percent.

When we use our model to simulate his faster phase-out, we find the following. First, our model simulates a smaller long-run increase in output from phasing out paygo. Roughly half of the difference in the output increase between our two models is due to differences in model design, and half to differences in parameter values. Second, our model estimates a somewhat larger loss for older persons and somewhat larger gains for younger persons. Most of the differences in rest-of-life utility are due to model design.

Table 5 provides information comparing the transition path of our model with Kotlikoff's [1998]. Each column shows the percentage change in output and in the interest rate  $T$  years after the enactment of the phase-out of paygo Social Security. The numbers in column 1 are reported by Kotlikoff [1998, Tables 7.5 and 7.7, row 11]. Column 2 gives the results of our model using Kotlikoff's parameter values, delay, and speed of phase-out. Column 3 gives the results of our model using our base parameter values, but Kotlikoff's delay and speed of phase-out. The steady-state interest rate with paygo is similar in all three columns: 9.1 percent for column 1, 9.0 percent for column 2, and 8.1 percent for column 3.

Consider the 150-year row in the output (top) block of Table 5. Column 1 shows that in Kotlikoff's model, phasing out paygo raises output 17.1 percent. Column 2 shows that in our model with Kotlikoff's parameters, phasing out paygo raises output 13.1 percent. Column 3 shows that in our model with our base parameters but with

**TABLE 6**  
**Model Comparisons:**  
**Percentage Change in Rest-Of-Life Gains (+) and Losses (–)**  
**55-Year Paygo Phase-Out:  $t_p$  Delay=10,  $t_p$  Duration=45**

Age	Kotlikoff [1998]	Kotlikoff Parameter Values <sup>a</sup>	Base Parameter Values <sup>b</sup>
74	0.0%	0.0%	0.0%
45	–1.2%	–2.7%	–3.5%
30	–0.4%	–1.2%	–1.7%
20	0.3%	0.3%	0.1%
10	1.9%	3.6%	3.5%

a. Kotlikoff Parameter Values:  $n = 1$  percent,  $g = 0$  percent,  $\alpha = 0.25$ ,  $\sigma = 0.25$ ,  $\rho = 1.5$  percent, Last Working Age = 65 ( $R=45$ ), Last Retirement Age = 75 ( $J = 55$ ).

b. Base Parameter Values:  $n = 0$  percent,  $g = 2$  percent,  $\alpha = 0.3$ ,  $\sigma = 0.5$ ,  $\rho = 1.5$  percent, Last Working Age = 65 ( $R = 45$ ), Last Retirement Age = 80 ( $J = 60$ ).

Kotlikoff's delay and speed of phase-out, phasing out paygo raises output 9.8 percent. Thus, our model simulates a smaller impact on output from the phasing out of paygo—roughly half the difference is due to model design, and half to parameter values.

Consider the 150 year row in the interest rate (bottom) block of Table 5. Column 1 shows that in Kotlikoff's model, phasing out paygo reduces the interest rate 25.3 percent (from 9.1 percent to 6.8 percent). Column 2 shows that in our model with Kotlikoff's parameters, phasing out paygo reduces the interest rate 30.9 percent (from 9.0 percent to 6.2 percent). Column 3 shows that in our model with our base parameters but with Kotlikoff's delay and speed of phase-out, phasing out paygo reduces the interest rate 19.6 percent (from 8.1 percent to 6.5 percent). Thus, the beginning and ending interest rates are similar in all three versions of the models.

Table 6 provides information comparing cohort welfare along the transition path of our model with Kotlikoff's [1998]. Each column shows the percentage change in rest-of-life welfare of particular age cohorts. As in Table 5, the numbers in column 1 are reported by Kotlikoff [1998, Table 7.8, row 11]; column 2 gives the results of our model using Kotlikoff's parameter values, delay, and speed of phase-out; and column 3 gives the results of our model using our base parameter values, but Kotlikoff's delay and speed of phase-out.

In Table 6, the welfare gains and losses vary somewhat across the three columns. Consider a person who is age 45 in the year of enactment. Column 1 shows that in Kotlikoff's model, phasing out paygo reduces rest-of-life welfare 1.2 percent; column 2 shows that in our model with Kotlikoff's parameters, the reduction is 2.7 percent; and column 3 shows that in our model with our base parameters but with Kotlikoff's delay and speed of phase-out, the reduction is 3.5 percent. Thus, our model estimates a somewhat larger loss for persons age 45—roughly two-thirds of the difference is due to model design, and one-third due to parameter values.

Now consider a person who is age 10 in the year of enactment. Column 1 shows that in Kotlikoff's model, phasing out paygo increases rest-of-life welfare 1.9 percent; column 2 shows that in our model with Kotlikoff's parameters, the increase is 3.6 percent; and column 3 shows that in our model with our base parameters but with

Kotlikoff's delay and speed of phase-out, the increase is 3.5 percent. Thus, our model estimates a somewhat larger gain for persons age 10—all due to the difference in model design.

## CONCLUSIONS

This paper makes a contribution to providing the quantitative information that is necessary for evaluating whether to gradually convert pay-as-you-go (paygo) Social Security to a funded system. It uses a life-cycle growth model, with empirically-plausible parameter values based on econometric studies, to investigate the transition path from paygo to full funding. It differs from earlier studies in focusing on the sensitivity of the magnitude of losses and gains of particular age cohorts to the speed of phasing out paygo Social Security and to plausible changes in key parameter values. For each age cohort we compute the gain in rest-of-life utility that results from conversion.

For our base parameter values, we find that as the transition evolves, the gain approaches 19.2 percent for cohorts born further into the future. Hence, each person born in the distant future will be 19.2 percent better off if the economy has funded Social Security than if the economy has paygo Social Security. Below we provide information on how long it takes for the gain to approach its steady-state value of 19.2 percent. With funded Social Security, a worker's rate of return on Social Security saving equals the marginal product of capital, whereas with paygo, the return equals the sum of labor force and productivity growth; with our base parameter values, the funded return is 6.5 percent, whereas the paygo return is only 2 percent. To achieve the same 45 percent replacement rate, funded Social Security requires only a 3.6 percent payroll tax rate, whereas paygo Social Security requires a 15 percent rate. Thus, converting paygo to funded Social Security eventually achieves a better steady-state. However, the transition to that better steady-state involves losses for particular age cohorts. We perform simulations with our model to estimate the magnitude and pattern of these losses.

We begin by considering a 90-year (three-generation) phase-out, and investigate the consequence of beginning the phase-out immediately, or delaying it either 10 or 15 years. With a 15-year delay, the maximum loss for any age cohort is 1.6 percent (for a person age 43 in the year of enactment). An older worker age 60 experiences only a very small loss (−0.4 percent), a younger worker age 30 experiences a small loss (−1.1 percent), and the younger worker's new-born child age 0 enjoys a rest-of-life gain of 2.9 percent. Without the delay, the maximum loss for any age cohort would be 3.3 percent instead of 1.6 percent, but the gain for a new-born would be 5.4 percent instead of 2.9 percent.

We focus our attention on four phase-out options (all with 15-year delays): 45, 60, 75, and 90 years. The advantage of the slower 90-year phase-out over the faster phase-outs (75, 60, and 45) is that the maximum loss for any age is smaller (1.6 percent vs 2.0 percent, 2.8 percent, 4.2 percent). The disadvantage of the slower 90-year phase-out is that the gain for a newborn is also smaller (2.9 percent vs 3.7 percent, 5.8 percent, and 9.2 percent; recall that in the distant future a person will be 19.2 percent better off).



We consider empirically-plausible variations in each parameter of the model, and find that our conclusions concerning the pattern of cohort gains and losses are not sensitive to these variations. Finally, we compare our results with Kotlikoff's [1998]. In our model, it is necessary to convert more slowly than Kotlikoff, 90 years instead of his 55 years, in order to keep the maximum loss the same as he reports—roughly 2 percent.

## APPENDIX

Throughout this appendix we will indicate  $\sum_L^U X^i$  by the slightly simpler notation  $M_L^U[X]$ , so  $M_L^U[X] = (X^{U+1} - X^L)/(X - 1)$  for  $X \neq 1$ , and  $M_L^U[X] = U + 1 - L$  for  $X = 1$ .

### *Steady States*

The following derivation applies to both paygo and funded steady states. For the paygo steady state, the paygo payroll tax rate  $t_p > 0$ ; for the funded steady state,  $t_p = 0$ . The funded payroll tax rate  $t_f$  has no effect on the funded steady state, and is only relevant for computing the funded replacement rate.

Population age  $t$  in year  $v$  is  $(1 + n)^{v-t}$ . Each person is “born” at age 0, enters the economy at “model age” 1, works from ages 1 through  $R$ , and retires from ages  $R + 1$  through  $J$  (for example, if a person enters the economy after “real-life age” 20 and retires after age 65 through age 80, then  $R = 45$  and  $J = 60$ ). The wage in the economy when a person is age  $t$  is  $w_t = w_0(1 + g)^t$ , where  $w_0$  is the wage when the person was age 0. In any year  $v$ , revenue from workers,  $t_p w [\sum_1^R (1 + n)^{v-t}]$ , equals paygo benefits to retirees,  $B_p [\sum_{R+1}^J (1 + n)^{v-t}]$ , or  $B_p/w = t_p \omega$ , where  $\omega$  is the worker/retiree ratio.<sup>23</sup> We show<sup>24</sup> that  $r = n + g + ng$ . Each entrant chooses  $C_t$  ( $t = 1, \dots, J$ ) to maximize utility function (A1) subject to budget constraint (A2):

$$(A1) \quad U = \sum_1^J [(C_t^{-\gamma} - 1)/(1 - \gamma)]/(1 + \rho)^t, \quad \gamma > 0, (1 + \rho) > 0, \text{ if } \gamma \neq 1, \text{ or}$$

$$U = \sum_1^J \ln C_t / (1 + \rho)^t, \text{ if } \gamma = 1.$$

$$(A2) \quad \sum_1^J C_t / (1 + r)^t = \sum_1^R (1 - t_p) w_t / (1 + r)^t + \sum_{R+1}^J B_{pt} / (1 + r)^t.$$

We obtain<sup>25</sup>  $C_t = (1/\lambda)^\sigma [(1 + r)/(1 + \rho)]^{\sigma t}$ , where  $\sigma \equiv 1/\gamma$  and  $\lambda$  is the Lagrange multiplier; hence, at entry the person plans a lifetime consumption path with a constant growth rate equal to  $[(1 + r)/(1 + \rho)]^\sigma - 1$ ; in the steady state this consumption plan will be realized. For a person who is age  $t$  in year  $v$ , we find that<sup>26</sup>  $C_t(v) = w_e(1 + g)^{v-t} \{ (1 - t_p) M_1^R [(1 + g)/(1 + r)] + t_p \omega M_{R+1}^J [(1 + g)/(1 + r)] \} [(1 + r)/(1 + \rho)]^{\sigma t} / M_1^J [(1 + r)^{\sigma-1} / (1 + \rho)^\sigma]$ . Consumption by the cohort age  $t$  in year  $v$  is  $(1 + n)^{v-t} C_t(v)$ , so aggregate consumption is  $C(v) = \sum_1^J (1 + n)^{v-t} C_t(v) = (1 + n)^v w_e (1 + g)^v \{ (1 - t_p) M_1^R [(1 + g)/(1 + r)] + t_p \omega M_{R+1}^J [(1 + g)/(1 + r)] \} M_1^J [z] / M_1^J [(1 + r)^{\sigma-1} / (1 + \rho)^\sigma]$ , where  $z \equiv [(1 + r)/(1 + \rho)]^\sigma / (1 + n)$  ( $1 + g$ ). The ratio of  $C(v)$  to  $w(v)L(v)$  is<sup>27</sup>

$$\begin{aligned}
 (A3) \quad C/wL &= \{(1 - t_p)M_1^R[(1 + g)/(1 + r)] \\
 &\quad + t_p \omega M_{R+1}^J[(1 + g)/(1 + r)]\} M_1^J[z] \\
 &\quad / M_1^J[(1 + r)^{\sigma-1}/(1 + \rho)^\sigma] M_1^R[1/(1 + n)].
 \end{aligned}$$

Aggregate saving<sup>28</sup> is  $S = wL + rK - C$ ; in the steady state, the growth rate of capital,  $S/K$ , equals the growth rate of effective labor, so  $S/K = (n + g + ng)$ ; combining these two equations yields

$$(A4) \quad K/wL = \{[C/wL] - 1\} / (r - n - g - ng).$$

From the marginal productivity relationships<sup>29</sup>,  $w_e = (1 - \alpha)k^\alpha$  and  $r = \alpha/k^{1-\alpha}$ , so

$$(A5) \quad K/wL = \alpha/(1 - \alpha)r.$$

Substituting (A3) into (A4) and combining with (A5), we obtain

$$\begin{aligned}
 (A6) \quad r &= \{[\alpha/(1 - \alpha)](r - n - g - ng)\} / \{[(1 - t_p)M_1^R[(1 + g)/(1 + r)] \\
 &\quad + t_p \omega M_{R+1}^J[(1 + g)/(1 + r)]\} M_1^J[z] / M_1^J[(1 + r)^{\sigma-1}/(1 + \rho)^\sigma] M_1^R[1/(1 + n)]\} - 1,
 \end{aligned}$$

where  $z \equiv [(1 + r)/(1 + \rho)]^\sigma / (1 + n)(1 + g)$ . Note that  $r$  is the only endogenous variable in (A6) and appears on both sides; for any set of parameter values,  $r$  is obtained by an iterative procedure.

Let  $C_t$  be the person's consumption path in the funded steady state,  $\bar{C}_t$  be the consumption path in the paygo steady state, and  $\delta$  be the percentage increase in the paygo path  $\bar{C}_t$  that would achieve the same utility as the person achieves with the funded path  $C_t$ .  $\delta$  is defined by

$$(A7) \quad \sum_1^J \{[(1 + \delta)C_t]^{1-\gamma} - 1\} / (1 - \gamma)(1 + \rho)^t = \sum_1^J (C_t)^{1-\gamma} / (1 - \gamma)(1 + \rho)^t.$$

It follows that

$$(A8) \quad \delta = \{[\sum_1^J (C_t)^{1-\gamma} / (1 + \rho)^t] / [\sum_1^J (\bar{C}_t)^{1-\gamma} / (1 + \rho)^t] - 1\}.$$

In the funded steady state, for each person the present value of fund benefits received as a retiree equals the present value of fund taxes paid as a worker, so  $\sum_1^R t_f w_{\rho_0} (1 + g)^t / (1 + r)^t = \sum_{R+1}^J B_{ft} / (1 + r)^t$ , where the benefit at age  $t$  required to maintain the paygo replacement rate is  $B_{ft} = t_p \omega w_t = t_p \omega w_{\rho_0} (1 + g)^t$ . This yields the following formula for the required  $t_f$ :

$$(A9) \quad t_f = \{\omega M_{R+1}^J[(1 + g)/(1 + r)] / M_1^R[(1 + g)/(1 + r)]\} t_p.$$

The surplus per effective labor is

$$(A10) \quad [su]_f \equiv t_f w_e + rk_f - t_p w_e.$$

All terms are “per effective labor”:  $t_f w_e$  is the fund payroll tax and  $rk_f$  is the fund interest income;  $t_p w_e$  is the fund benefit because  $t_p w_{pe}$  is the tax and benefit in the paygo steady state ( $w_{pe}$  is the paygo wage) and to maintain the paygo replacement rate with the funded Social Security wage  $w_e$ ,  $t_p w_{pe}$  must be multiplied by  $(w_e/w_{pe})$ , yielding benefit  $t_p w_e$ . In the steady state,  $[su]_f/k_f = n + g + ng$ . Substituting into (A10), dividing by  $k$ , yields

$$(A11) \quad k_f/k = (t_p - t_f)r/[\alpha/(1 - \alpha)](r - n - g - ng),$$

where  $k_f$  is Social Security capital per effective labor.

### Transition Paths

We begin in the paygo steady state in year  $v = 0$ . We compute  $H_t(0)$ , the wealth a person of age  $t$  in year  $v = 0$  has accumulated in the paygo steady state.<sup>30</sup> Once conversion occurs and the simulation begins, for each person,  $H_{t+1}(v+1) = H_t(v) + S_t(v)$  where  $H_t$  is the person's wealth (divided between his funded Social Security account  $F$  and his private account  $A$ ) and  $S_t$  is the person's saving (channeled into his two accounts). We assume the division between the two accounts has no effect on the individual or the economy; it does, however, determine the funded Social Security replacement rate, as shown below. In each year  $v$ , a person of age  $t$  chooses a rest-of-life path  $C_i$  ( $i = t, \dots, J$ ) to maximize rest-of-life utility (A12) subject to the rest-of-life budget constraint (A13):

$$(A12) \quad U = \sum_{i=t}^J [(C_i^{1-\gamma} - 1)/(1 - \gamma)]/(1 + \rho)^{i-t}, \gamma > 0, (1 + \rho) > 0, \text{ if } \gamma \neq 1, \text{ or}$$

$$U = \sum_{i=t}^J \ln C_i / (1 + \rho)^{i-t}, \text{ if } \gamma = 1,$$

$$(A13) \quad \sum_{i=t}^J \{C_i / \Pi_{j=1}^{i-t} (1 + r_j)\} = H_t(1 + r) + Z \sum_{i=t}^R \{(1 - t_p[i - t + v]) w \Pi_{j=1}^{i-t} (1 + \hat{w}_j)$$

$$/ \Pi_{j=1}^{i-t} (1 + r_j)\} + \sum_{i=u}^J \{t_p[i - t + v] w \Pi_{j=1}^{i-t}$$

$$(1 + \hat{w}_j) / \Pi_{j=1}^{i-t} (1 + r_j)\},$$

where  $H \equiv A + F$ ;  $Z = 1$  if  $t \leq R$ ,  $Z = 0$  if  $t \geq R + 1$ ,  $u = R + 1$  if  $t \leq R$ ,  $u = t$  if  $t \geq R + 1$ ,  $r$  is the current interest rate,  $w$  is the current wage,  $t_p[i - t + v]$  is the rest-of-life path of the wage tax rate,  $\hat{w}_j$  is the growth rate of the wage  $j$  years in the future, and  $r_j$  is the interest rate  $j$  years in the future. Note that the product  $\Pi$  is over  $j$  (for  $i = t$ ,  $\Pi = 1$ ), and the sum  $\Sigma$  is over  $i$ . We assume each individual has foresight concerning the paths  $t_p[i - t + v]$ ,  $\hat{w}_j$ , and  $r_j$ . Using the Lagrange multiplier method for a constrained maximization:

$$\begin{aligned}
U^* &= \Sigma_t^J [(C_i^{1-\gamma} - 1)/(1 - \gamma)]/(1 + \rho)^{i-t} \\
&+ \lambda [H_t(1 + r) + Z \Sigma_t^R \{[1 - t_p(i - t + v)] w \Pi_1^{i-t}(1 + \hat{w}_j)/\Pi_1^{i-t}(1 + r_j)\} \\
&+ \Sigma_u^J [t_p(i - t + v) \omega w \Pi_1^{i-t}(1 + \hat{w}_j)/\Pi_1^{i-t}(1 + r_j)] - \Sigma_t^J [C_i/\Pi_1^{i-t}(1 + r_j)]] \\
\partial U^*/\partial C_i &= [C_i^{-\gamma}/(1 + \rho)^{i-t}] - [\lambda/\Pi_1^{i-t}(1 + r_j)] = 0,
\end{aligned}$$

so

$$(A14) \quad C_i/\Pi_1^{i-t}(1 + r_j) = (1/\lambda)^\sigma [\Pi_1^{i-t}(1 + r_j)]^{\sigma-1}/(1 + \rho)^{\sigma(i-t)}.$$

Substituting (A14) into (A13) yields

$$\begin{aligned}
(A15) \quad (1/\lambda)^\sigma &= [H_t(1 + r) + Z \Sigma_t^R \{[1 - t_p(i - t + v)] w \Pi_{j=1}^{i-t}(1 + \hat{w}_j)/\Pi_1^{i-t}(1 + r_j) \\
&+ \Sigma_u^J t_p(i - t + v) \omega w \Pi_1^{i-t}(1 + \hat{w}_j)/\Pi_1^{i-t}(1 + r_j)\}] \\
&/ (1 + \rho)^{\sigma t} \{\Sigma_t^J [\Pi_1^{i-t}(1 + r_j)]^{\sigma-1}/(1 + \rho)^{\sigma i}\}.
\end{aligned}$$

Substituting (A15) into (A14), and setting  $i = t$ , yields

$$\begin{aligned}
(A16) \quad C_t(v) &= [H_t(1 + r) + Z \Sigma_{i=t}^R \{[1 - t_p(i - t + v)] w \Pi_{j=1}^{i-t}(1 + \hat{w}_j)/\Pi_{j=1}^{i-t}(1 + r_j)\} \\
&+ \Sigma_{i=u}^J t_p(i - t + v) \omega w \Pi_{j=1}^{i-t}(1 + \hat{w}_j)/\Pi_{j=1}^{i-t}(1 + r_j)] \\
&/ (1 + \rho)^{\sigma t} \{\Sigma_{i=t}^J [\Pi_{j=1}^{i-t}(1 + r_j)]^{\sigma-1}/(1 + \rho)^{\sigma i}\},
\end{aligned}$$

where  $r$  is the current interest rate,  $Z = 1$  if  $t \leq R$ ,  $Z = 0$  if  $t \geq R + 1$ ,  $u = R + 1$  if  $t \leq R$ ,  $u = t$  if  $t \geq R + 1$ ,  $w$  is the current wage,  $t_p[i - t + v]$  is the rest-of-life path of the paygo wage tax,  $\hat{w}_j$  is the growth rate of the wage  $j$  years in the future, and  $r_j$  is the interest rate  $j$  years in the future. Note that the product  $\Pi$  is over  $j$  (for  $i = t$ ,  $\Pi = 1$ ), and the sum  $\Sigma$  is over  $i$ .

Then the saving of a person age  $t$  is  $S_t(v) = Z(1 - t_p)w + rH_t + (1 - Z)B_p - C_t$ . Aggregate saving is  $S(v) = \Sigma_1^J (1 + n)^{v-t} S_t(v) = (1 + n)^v \Sigma_1^J [S_t(v)/(1 + n)^t]$ ;  $E(v) = (1 + g)^v L(v)$ ;  $L(v) = \Sigma_1^R (1 + n)^{v-t} = (1 + n)^v \Sigma_1^R [1/(1 + n)]$ . The capital stock in year  $v + 1$  is  $K(v + 1) = K(v) + S(v)$ . Capital per effective labor in year  $v + 1$  is  $k(v + 1) \equiv K(v + 1)/E(v + 1)$ . Since each factor is paid its marginal product,  $w_e = (1 - \alpha)k^\alpha$  and  $r = \alpha k^{1-\alpha}$ , so from  $k(v + 1)$  we obtain  $r(v + 1)$  and  $w_e(v + 1)$ , where  $w_e$  is the wage per effective labor; the current wage per worker in the economy  $w(v + 1) = (1 + g)^v w_e(v + 1)$ .

A person's saving  $S$  is channeled into two accounts, the funded Social Security account  $F$  and a private account  $A$ ; both accounts earn  $r = \text{mpk}$ . The allocation of a person's saving between the two accounts,  $F$  and  $A$ , determines the funded Social Security replacement rate. The allocation to  $F$  is determined by the funded payroll tax rate  $t_f$  (the rest is allocated to  $A$ ). For each worker,  $F_t(0) = 0$ ,  $F_{t+1}(v + 1) =$

$[1 + r(v)]F_t(v) + t_f(v)w(v)$ , where the wage per worker  $w(v) = w_e(1 + g)^v$ , and  $t_f(v)$  is scheduled by policy makers. For a retiree,  $F_{t+1}(v + 1) = [1 + r(v)]F_t(v) - B_{\bar{t}}(v)$ , where the benefit  $B_{\bar{t}}(v)$  is set each year so that if it grows at rate  $g$ , the person's  $F$  fund will be exhausted at the end of retirement.<sup>31</sup> Then  $A_{t+1}(v + 1) = H_{t+1}(v + 1) - F_{t+1}(v + 1)$ . The aggregate fund  $F(v)$  is obtained by summing  $F_t(v)$  over the population. Each year the fund benefit for a retiree age  $t$ ,  $B_{\bar{t}}(v)$ , is set so  $\sum_t^J B_{\bar{t}}(v)(1 + g)^{i-t}/(1 + r)^{i-t} = (1 + r)F_t(v)$ , so  $B_{\bar{t}}(v) = (1 + r)F_t(v)/[(1 + r)/(1 + g)]^t M_t^J[(1 + g)/(1 + r)]$ . In year  $v$ , the total fund is  $\sum_1^J (1 + n)^{v-t} F_t(v)$ ; the total inflow to the fund is  $[t_f(v)w(v)\sum_1^R (1 + n)^{v-t} + r(v)\sum_1^J (1 + n)^{v-t} F_t(v)]$ , and total benefits paid are  $\sum_{R+1}^J (1 + n)^{v-t} B_{\bar{t}}(v)$ .

## NOTES

1. Below we discuss the differences between individual funds and a collective fund.
2. The superiority of the funded Social Security steady state over the paygo steady state may depend on the empirically-plausible assumption that the saving rate in both economies does not exceed the golden rule saving rate.
3. Feldstein and Samwick use data and forecasts for the U.S. economy, but they assume that wages and interest rates are unaffected by the capital accumulation that results from funding. They assume a real rate of return of 9 percent. Their simulation assumes that Social Security must pay benefits earned by taxes paid in the past under paygo, while simultaneously accumulating an aggregate fund that will eventually replace paygo financing. In their simulation, the payroll tax rate is raised for roughly two decades, but the increase is surprisingly modest. Beginning at 12.4 percent, the combined payroll tax rate is immediately raised 2 percentage points to 14.4 percent, its peak, and then is gradually phased down. It drops back to 12.4 percent in two decades, 9 percent in three decades, 6 percent in four decades, and converges to a permanent value of only 2 percent. Middle age and older workers are a bit worse off because the tax rate is a bit higher than 12.4 percent for about two decades. Young workers, however, are better off because the much lower tax rate after two decades outweighs the slightly higher tax rate in the first two decades. All future workers are much better off as the tax rate phases down to 2 percent.
4. He finds a maximum loss of 2 percent of rest-of-life utility. In the simulation, there is an immediate reduction in the payroll tax to zero and a schedule that gradually phases down the paygo Social Security benefit over many years. Most simulations have a schedule with a 10 year delay to protect current retirees and then a 45-year benefit phase-down so that each age cohort receives roughly the benefit to which it is entitled due to the payroll tax it has previously paid. Three alternative ways to finance these benefits during the phase-down are simulated: an income tax, a consumption tax, and borrowing. Kotlikoff reports that in his base case with transitional income tax financing, the utility (measured in wealth equivalents) of those born in the long run is about 10 percent higher than with paygo Social Security, while those in the economy at conversion experience a small rest-of-life loss.
5. They find a maximum loss of 2 percent of rest-of-life utility. As in Kotlikoff [1996; 1998], the simulations have a schedule with a 10-year delay and then a 45-year phase-down. They report that funding would yield welfare gains for all future workers. In the long run, utility (measured in wealth equivalents) would rise 8 percent for average earners, 6 percent for the poorest agents, and 4.4 percent for the richest agents. The loss for those in the economy at conversion, and the pattern of loss across earning classes in a given age cohort, depend on the method of financing the transitional paygo benefits.
6. Kotlikoff [1998] does consider the sensitivity of efficiency gains to the intertemporal elasticity of substitution, but he does not show the sensitivity of the pattern of cohort gains and losses.
7. If we included other government spending and taxes, the result would be a lower steady state capital per effective labor and a higher steady state marginal product of capital (mpk). Starting from a higher mpk economy, a switch to funding would probably generate lower short-run losses and greater long-run gains. Thus, by omitting other taxes, our study probably overstates short-run losses and understates long-run gains.
8. We assume exogenous retirement. Note, for example, with each person entering the economy after age 20, and retiring after age 65 through age 80, if  $n = 0$  percent,  $\omega = 3$ .

9. We assume an exogenous labor supply, so our model does not capture the elimination of the distortion in labor supply that results from replacing paygo with funded Social Security. In this respect, our study overstates short-run losses and understates long-run gains.
10. It should be emphasized that the model assumes a closed economy, perfect capital markets, constant wage growth at the rate of technical progress, and an absence of intra-family transfers.
11. The rate of return  $r$  that a worker receives on paygo Social Security is the interest rate that makes the present value of taxes equal the present value of benefits. Note that  $r$  is independent of  $t_p$  and of the age and length of retirement.
12. In the Appendix it is shown that this consumption growth rate equals  $[(1+r)/(1+\rho)]^\sigma - 1$ ; since the funded  $r$  (6.5 percent) is less than the paygo  $r$  (8.1 percent), the growth rate in the funded steady state (2.4 percent) is less than in the paygo steady state (3.2 percent).
13. A formula for the required tax rate with funding  $t_f$  is derived in the Appendix.
14. With the above parameter values (including  $t_p = 15$  percent), the share of the capital stock that is under Social Security (either in privatized individual accounts or in a single aggregate account),  $k_f/k$ , is 38 percent; the formula for  $k_f/k$  is derived in the Appendix. The annual fund inflow—payroll tax revenue plus fund interest income—that is not used to pay benefits equals the fund's annual surplus. With the above parameter values, it turns out that tax revenue is 18 percent, and fund interest, 82 percent, of the annual inflow; while benefits are 75 percent, and surplus, 25 percent, of the annual outflow; this surplus enables capital in the fund to grow at the same rate as capital, and effective labor, in the economy, so that the Social Security fund maintains a constant share of the capital of the economy, and the ratio of fund capital to effective labor remains constant.
15. The computer simulations do in fact converge to the steady-state values derived above.
16. Just as deposits into, and withdrawals from, the  $A$  account do not appear in a person's rest-of-life budget constraint, so deposits into and withdrawals from the  $F$  account do not appear. The person chooses a consumption path based on total wealth  $H = A + F$ .
17. In related simulations with tax conversion in a life-cycle model, we have examined two alternatives: perfect foresight [Lewis and Seidman, 2001] and steady-state expectations [Seidman and Lewis, 1999].
18. Note that in the text and in all tables, we always report the actual age, not the "model age." For example, we report the age of the oldest cohort in our economy as 80 (for our base case) in the text or a table; this corresponds to a "model age" of 60 ( $J = 60$ ). We report the age 0 for a newborn who will enter the labor force 20 years later (model age-20). Moreover, whenever we refer to the age of a cohort in the text or in the tables, we mean the person's actual age in year 0 when the scheduled phase-down is enacted. For example, a cohort with age 65 is a cohort that is about to retire in the year that the phase-down is enacted (year 0).
19. Because of rounding error, we define a loss as  $\delta < -0.05$  percent.
20. This statement should not be misinterpreted. Recall that we assume that each individual's utility depends only on that individual's own consumption path. Thus, intergenerational altruism is not included in the utility function.
21. Kotlikoff [1996; 1998] and Kotlikoff, Smetters and Walliser [1998] simulate a 55-year phase-out and find a maximum loss of about 2 percent, which is a bit lower than the 2.8 percent maximum loss that we find for our 60-year phase-out.
22. A range of baseline values for  $\sigma$  have been selected by different researchers: Laibson, Repetto, and Tobacman [1998],  $\sigma = 1.00$ ; Gale [1998],  $\sigma = 0.33$  or less; Kotlikoff, Smetters, and Walliser [1998],  $\sigma = 0.25$ ; Barsky, Juster, Kimball, and Shapiro [1997],  $\sigma = 0.18$ ; Carroll [1997],  $\sigma = 0.50$ ; Engen, Gravelle, and Smetters (1997),  $\sigma = 0.25$ ; Beaudry and van Wincoop [1996],  $\sigma = 1.00$ ; Auerbach [1997],  $\sigma = 0.25$ ; Engen and Gale [1996],  $\sigma = 0.33$ ; Hubbard, Skinner, and Zeldes [1995],  $\sigma = 0.33$ ; Engen, Gale, Scholz [1994],  $\sigma = 0.33$ ; Blanchard and Fischer [1993],  $\sigma \leq 1.00$ ; Attanasio and Weber [1993],  $0.48 < \sigma < 0.74$ ; Campbell and Mankiw [1989],  $\sigma = 0.28$ ; Hall [1988],  $\sigma < 0.20$ ; Auerbach and Kotlikoff [1987],  $\sigma = 0.25$ ; Hubbard and Judd [1986],  $0.10 < \sigma < 1.10$ ; Evans [1983],  $0.2 < \sigma < 1.0$ . Deaton [(1992)] provides a review and critique of empirical studies that attempt to estimate the intertemporal elasticity of substitution.
23. In year  $v$  the number of workers equals  $\sum_1^R (1+n)^{v-t} = (1+n)^v M_1^R [1/(1+n)]$ , the number of retirees equals  $\sum_{R+1}^J (1+n)^{v-t} = (1+n)^v M_{R+1}^J [1/(1+n)]$ , and  $\omega = M_1^R [1/(1+n)] / M_{R+1}^J [1/(1+n)]$ .
24. The rate of return  $r$  that a worker receives on paygo Social Security is defined implicitly by  $\sum_1^R t_p w_t / (1+r)^t = \sum_{R+1}^J B_{pt} / (1+r)^t$ , where  $B_{pt} = t_p \omega w_0 (1+g)^t$ . Substituting for  $w_t$ ,  $B_{pt}$ , and  $\omega$ , this equation is satisfied by  $(1+r) = (1+n)(1+g)$ , so  $r = n + g + ng$ .
25. Let  $U^* = \sum_1^J [(C_1^{1-\gamma} - 1)/(1-\gamma)] / (1+\rho)^t + \lambda [\sum_1^R w_t / (1+r)^t + \sum_{R+1}^J B_{pt} / (1+r)^t - \sum_1^J C_t / (1+r)^t]$ ;  $\partial U^* / \partial C_t = [C_t^{-\gamma} / (1+\rho)^t] - [\lambda / (1+r)^t] = 0$ , so  $C_t = (1/\lambda)^\sigma [(1+r)/(1+\rho)]^{\sigma t}$ , where  $\sigma \equiv 1/\gamma$ .



26. Substituting  $C_t = (1/\lambda)^\sigma [(1+r)/(1+\rho)]^{\sigma t}$  into (A2) yields  $(1/\lambda)^\sigma = w_0 \{ (1 - t_p) M_1^R [(1+g)/(1+r)] + t_p \omega M_{R+1}^J [(1+g)/(1+r)] \} / M_1^J [(1+r)^{\sigma-1}/(1+\rho)^\sigma]$ . Substituting for  $(1/\lambda)^\sigma$  in  $C_t = (1/\lambda)^\sigma [(1+r)/(1+\rho)]^{\sigma t}$ , using  $w_0 = w_e (1+g)^{v-t}$ , where  $w_e$  is the wage per effective labor, yields  $C_t(v)$ .
27. In year  $v$  all workers receive the same  $w(v) = w_e (1+g)^v$ , so aggregate labor income  $w(v)L(v) = \sum_1^R w_e (1+g)^v (1+n)^{v-t} = w_e (1+g)^v (1+n)^v M_1^R [1/(1+n)]$ .
28. With paygo Social Security, aggregate saving is  $S \equiv (1 - t_p)wL + rK + B_p N - C$  where  $N$  is the number of retirees; but  $t_p wL = B_p N$ , so  $S \equiv wL + rK - C$ .
29. The production function is  $y = k^\alpha$ , ( $0 < \alpha < 1$ ), so  $w_e = (1 - \alpha)k^\alpha$  and  $r = \alpha/k^{1-\alpha}$ . The saving rate of the economy is  $s \equiv S/Y$ ; in the steady state  $S/K = (n+g+ng)$ , so  $s = (n+g+ng)k^{1-\alpha}$ .
30. A person age  $t$  in year 0 was "born" in year  $v = -t$  when the wage in the economy was  $w_e (1+g)^{-t}$ . For each person in the economy in year 0 we compute the consumption path and simulate the wealth accumulation path of the person year by year to obtain  $A_t$  in year 0.
31. In the computer simulation, each person's fund is in fact exhausted at the end of retirement.

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